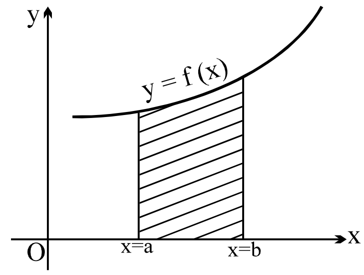
KEY CONCEPTS

THINGS TO REMEMBER :

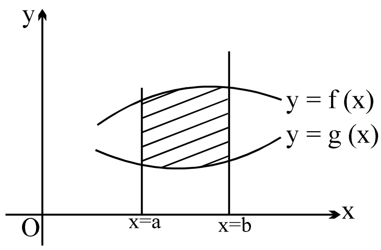


1. The area bounded by the curve $y = f(x)$, the x-axis and the ordinates at $x = a$ & $x = b$ is given by,

$$A = \int_a^b f(x)\,dx = \int_a^b y\,dx.$$

2. If the area is below the x–axis then A is negative. The convention is to consider the magnitude only i.e.

$$A = \left|\int_a^b y\;dx\right| \text{ in this case.}$$



3. Area between the curves $y = f(x)$ & $y = g(x)$ between the ordinates at $x = a$ & $x = b$ is given by,

$$A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b [\,f(x) - g(x)\,]\,dx.$$

4. Average value of a function $y = f(x)$ w.r.t. *x* over an interval $a \le x \le b$ is defined as :

$$y(av) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

5. The area function A_a^x satisfies the differential equation $\frac{dA_a^x}{dx} = f(x)$ with initial condition $A_a^a = 0$.

Note : If $F(x)$ is any integral of $f(x)$ then ,

$A_a^x = \int f(x)\,dx = F(x) + c$ $\qquad$ $A_a^a = 0 = F(a) + c \;\Rightarrow\; c = -F(a)$

hence $A_a^x = F(x) - F(a)$. Finally by taking $x = b$ we get , $A_a^b = F(b) - F(a)$.

6. **CURVE TRACING :**

The following outline procedure is to be applied in Sketching the graph of a function $y = f(x)$ which in turn will be extremely useful to quickly and correctly evaluate the area under the curves.

(a) Symmetry : The symmetry of the curve is judged as follows :

(i) If all the powers of *y* in the equation are even then the curve is symmetrical about the axis of *x*.

(ii) If all the powers of *x* are even , the curve is symmetrical about the axis of *y*.

(iii) If powers of *x* & *y* both are even, the curve is symmetrical about the axis of *x* as well as *y*.

(iv) If the equation of the curve remains unchanged on interchanging *x* and *y*, then the curve is symmetrical about $y = x$.

(v) If on interchanging the signs of *x* & *y* both the equation of the curve is unaltered then there is symmetry in opposite quadrants.

(b) Find dy/dx & equate it to zero to find the points on the curve where you have horizontal tangents.

(c) Find the points where the curve crosses the x–axis & also the y–axis.

(d) Examine if possible the intervals when $f(x)$ is increasing or decreasing. Examine what happens to 'y' when $x \to \infty$ or $-\infty$.

7. **USEFUL RESULTS :**

(i) Whole area of the ellipse, $x^2/a^2 + y^2/b^2 = 1$ is πab.

(ii) Area enclosed between the parabolas $y^2 = 4ax$ & $x^2 = 4by$ is $16ab/3$.

(iii) Area included between the parabola $y^2 = 4ax$ & the line $y = mx$ is $8a^2/3m^3$.

EXERCISE–I

Q.1 Find the area bounded on the right by the line $x + y = 2$, on the left by the parabola $y = x^2$ and below by the x – axis.

Q.2 Find the area of the region bounded by the curves, $y = x^2 + 2$; $y = x$; $x = 0$ & $x = 3$.

Q.3 Find the area of the region $\{(x, y) : 0 \le y \le x^2 + 1,\ 0 \le y \le x + 1,\ 0 \le x \le 2\}$.

Q.4 Find the value of c for which the area of the figure bounded by the curves $y = \sin 2x$, the straight lines $x = \pi/6$, $x = c$ & the abscissa axis is equal to 1/2.

Q.5 The tangent to the parabola $y = x^2$ has been drawn so that the abscissa x_0 of the point of tangency belongs to the interval [1, 2]. Find x_0 for which the triangle bounded by the tangent, the axis of ordinates & the straight line $y = x_0^2$ has the greatest area.

Q.6 Compute the area of the region bounded by the curves $y = e.x.\ln x$ & $y = \ln x/(e.x)$ where $\ln e=1$.

Q.7 A figure is bounded by the curves $y = \left|\sqrt{2}\sin\frac{\pi x}{4}\right|$, $y = 0$, $x = 2$ & $x = 4$. At what angles to the positive x–axis straight lines must be drawn through (4, 0) so that these lines partition the figure into three parts of the same size.

Q.8 Find the area of the region bounded by the curves, $y = \log_e x$, $y = \sin^4 \pi x$ & $x = 0$.

Q.9 Find the area bounded by the curves $y = \sqrt{1-x^2}$ and $y = x^3 - x$. Also find the ratio in which the y-axis divided this area.

Q.10 If the area enclosed by the parabolas $y = a - x^2$ and $y = x^2$ is $18\sqrt{2}$ sq. units. Find the value of 'a'.

Q.11 The line $3x + 2y = 13$ divides the area enclosed by the curve, $9x^2 + 4y^2 - 18x - 16y - 11 = 0$ into two parts. Find the ratio of the larger area to the smaller area.

Q.12 Find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ & $(x - 1)^2 + y^2 = 1$

Q.13 Find the values of m (m > 0) for which the area bounded by the line $y = mx + 2$ and $x = 2y - y^2$ is, (i) 9/2 square units & (ii) minimum. Also find the minimum area.

Q.14 Find the ratio in which the area enclosed by the curve $y = \cos x$ $(0 \le x \le \pi/2)$ in the first quadrant is divided by the curve $y = \sin x$.

Q.15 Find the area enclosed between the curves : $y = \log_e (x + e)$, $x = \log_e (1/y)$ & the x–axis.

Q.16 Find the area of the figure enclosed by the curve $(y - \arcsin x)^2 = x - x^2$.

Q.17 For what value of 'a' is the area bounded by the curve $y = a^2x^2 + ax + 1$ and the straight line $y = 0$, $x = 0$ & $x = 1$ the least ?

Q.18 Find the positive value of 'a' for which the parabola $y = x^2 + 1$ bisects the area of the rectangle with vertices (0, 0), (a, 0), $(0, a^2 + 1)$ and $(a, a^2 + 1)$.

Q.19 Compute the area of the curvilinear triangle bounded by the y–axis & the curve, $y = \tan x$ & $y = (2/3) \cos x$.

Q.20 Consider the curve $C : y = \sin 2x - \sqrt{3}\ |\sin x|$, C cuts the x – axis at (a, 0), $a \in (-\pi, \pi)$.
A_1 : The area bounded by the curve C & the positive x – axis between the origin & the ordinate at x = a.
A_2 : The area bounded by the curve C & the negative x – axis between the ordinate x = a & the origin.
Prove that $A_1 + A_2 + 8 A_1 A_2 = 4$.

Q.21 Find the area bounded by the curve $y = x e^{-x}$; $xy = 0$ and $x = c$ where c is the x-coordinate of the curve's inflection point.

Q.22 Find the value of 'c' for which the area of the figure bounded by the curve, $y = 8x^2 - x^5$, the straight lines $x = 1$ & $x = c$ & the abscissa axis is equal to 16/3.

Q.23 Find the area bounded by the curve $y^2 = x$ & $x = |y|$.

Q.24 Find the area bounded by the curve $y = x e^{-x^2}$, the x-axis, and the line x = c where y (c) is maximum.

Q.25 Find the area of the region bounded by the x–axis & the curves defined by,

$$\begin{cases} y = \tan x, & -\pi/3 \le x \le \pi/3 \\ y = \cot x, & \pi/6 \le x \le 3\pi/2 \end{cases}$$

EXERCISE–II

Q.1 In what ratio does the x-axis divide the area of the region bounded by the parabolas $y = 4x - x^2$ & $y = x^2 - x$?

Q.2 Find the area bounded by the curves $y = x^4 - 2x^2$ & $y = 2x^2$.

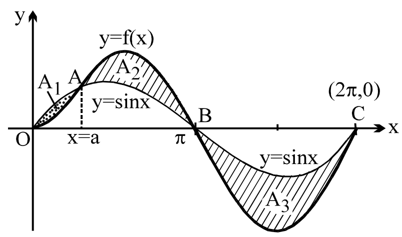
Q.3 Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ & $y = |x - 1|$ & find its area.

Q.4 Find the equation of the line passing through the origin and dividing the curvilinear triangle with vertex at the origin , bounded by the curves $y = 2x - x^2$, $y = 0$ and $x = 1$ into two parts of equal area.

Q.5 Consider the curve $y = x^n$ where $n > 1$ in the 1st quadrant. If the area bounded by the curve, the x-axis and the tangent line to the graph of $y = x^n$ at the point (1, 1) is maximum then find the value of n.

Q.6 Consider the collection of all curve of the form $y = a - bx^2$ that pass through the the point (2, 1), where a and b are positive constants. Determine the value of a and b that will minimise the area of the region bounded by $y = a - bx^2$ and x-axis. Also find the minimum area.

Q.7 In the adjacent graphs of two functions $y = f(x)$ and $y = \sin x$ are given. $y = \sin x$ intersects, $y = f(x)$ at A (a, f(a)); B(π, 0) and C(2π, 0). A_i (i = 1, 2, 3,) is the area bounded by the curves $y = f(x)$ and $y = \sin x$ between $x = 0$ and $x = a$; i = 1, between $x = a$ and $x = \pi$; i = 2, between $x = \pi$ and $x = 2\pi$; i = 3. If $A_1 = 1 - \sin a + (a - 1)\cos a$, determine the function f(x). Hence determine 'a' and A_1. Also calculate A_2 and A_3.



Q.8 Consider the two curves $y = 1/x^2$ & $y = 1/[4(x - 1)]$.
(i) At what value of 'a' ($a > 2$) is the reciprocal of the area of the fig. bounded by the curves, the lines $x = 2$ & $x = a$ equal to 'a' itself ?
(ii) At what value of 'b' ($1 < b < 2$) the area of the figure bounded by these curves, the lines $x = b$ & $x = 2$ equal to $1 - 1/b$.

Q.9 Show that the area bounded by the curve $y = \dfrac{\ln x - c}{x}$, the x-axis and the vertical line through the maximum point of the curve is independent of the constant c.

Q.10 For what value of 'a' is the area of the figure bounded by the lines, $y = \dfrac{1}{x}$, $y = \dfrac{1}{2x - 1}$, $x = 2$ & $x = a$ equal to $\ln \dfrac{4}{\sqrt{5}}$?

Q.11 Compute the area of the loop of the curve $y^2 = x^2 [(1 + x)/(1 - x)]$.

Q.12 Find the value of K for which the area bounded by the parabola $y = x^2 + 2x - 3$ and the line $y = Kx + 1$ is least. Also find the least area.

Q.13 Let A_n be the area bounded by the curve $y = (\tan x)^n$ & the lines $x = 0$, $y = 0$ & $x = \pi/4$. Prove that for $n > 2$, $A_n + A_{n-2} = 1/(n - 1)$ & deduce that $1/(2n + 2) < A_n < 1/(2n - 2)$.

Q.14 If f (x) is monotonic in (a, b) then prove that the area bounded by the ordinates at x = a ; x = b ; y = f (x) and y = f (c), c $\in$ (a, b) is minimum when $c = \dfrac{a + b}{2}$.
Hence if the area bounded by the graph of $f(x) = \dfrac{x^3}{3} - x^2 + a$, the straight lines x = 0, x = 2 and the x-axis is minimum then find the value of 'a'.

Q.15 Consider the two curves $C_1 : y = 1 + \cos x$ & $C_2 : y = 1 + \cos(x - \alpha)$ for $\alpha \in \left(0 , \dfrac{\pi}{2}\right)$; $x \in [0, \pi]$. Find the value of α, for which the area of the figure bounded by the curves C_1, C_2 & x = 0 is same as that of the figure bounded by C_2 , y = 1 & $x = \pi$. For this value of α, find the ratio in which the line y = 1 divides the area of the figure by the curves C_1, C_2 & $x = \pi$.

Q.16 Find the area bounded by $y^2 = 4(x + 1)$, $y^2 = -4(x - 1)$ & $y = |x|$ above axis of x.

Q.17 Compute the area of the figure which lies in the first quadrant inside the curve

$x^2 + y^2 = 3a^2$ & is bounded by the parabola $x^2 = 2ay$ & $y^2 = 2ax$ $(a > 0)$.

Q.18 Consider a square with vertices at $(1, 1), (-1, 1), (-1, -1)$ & $(1, -1)$. Let S be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S & find its area.

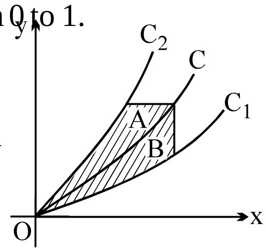
Q.19 Find the whole area included between the curve $x^2y^2 = a^2(y^2 - x^2)$ & its asymptotes (asymptotes are the lines which meet the curve at infinity).

Q.20 For what values of $a \in [0, 1]$ does the area of the figure bounded by the graph of the function $y = f(x)$ and the straight lines $x = 0$, $x = 1$ & $y = f(a)$ is at a minimum & for what values it is at a maximum if $f(x) = \sqrt{1 - x^2}$. Find also the maximum & the minimum areas.

Q.21 Find the area enclosed between the smaller arc of the circle $x^2 + y^2 - 2x + 4y - 11 = 0$ & the parabola $y = -x^2 + 2x + 1 - 2\sqrt{3}$.

Q.22 Draw a neat and clean graph of the function $f(x) = \cos^{-1}(4x^3 - 3x)$, $x \in [-1, 1]$ and find the area enclosed between the graph of the function and the x–axis as x varies from 0 to 1.

Q.23 Let C_1 & C_2 be two curves passing through the origin as shown in the figure. A curve C is said to "bisect the area" the region between C_1 & C_2, if for each point P of C, the two shaded regions A & B shown in the figure have equal areas. Determine the upper curve C_2, given that the bisecting curve C has the equation $y = x^2$ & that the lower curve C_1 has the equation $y = x^2/2$.



Q.24 For what values of $a \in [0, 1]$ does the area of the figure bounded by the graph of the function $y = f(x)$ & the straight lines $x = 0$, $x = 1$, $y = f(a)$ have the greatest value and for what values does it have the least value, if, $f(x) = x^\alpha + 3x^\beta$, $\alpha, \beta \in R$ with $\alpha > 1$, $\beta > 1$.

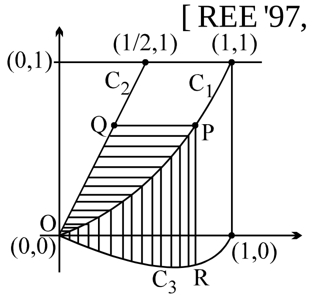
Q.25 Given $f(x) = \int_0^x e^t(\log\sec t - \sec^2 t)\,dt$; $g(x) = -2e^x \tan x$. Find the area bounded by the curves $y = f(x)$ and $y = g(x)$ between the ordinates $x = 0$ and $x = \frac{\pi}{3}$.

EXERCISE–III

Q.1 Let $f(x) = \text{Maximum}\{x^2, (1 - x)^2, 2x(1 - x)\}$, where $0 \le x \le 1$. Determine the area of the region bounded by the curves $y = f(x)$, x– axis, $x = 0$ & $x = 1$. [JEE '97, 5]

Q.2 Indicate the region bounded by the curves $x^2 = y$, $y = x + 2$ and x–axis and obtain the area enclosed by them. [REE '97, 6]

Q.3 Let C_1 & C_2 be the graphs of the functions $y = x^2$ & $y = 2x$, $0 \le x \le 1$ respectively. Let C_3 be the graph of a function $y = f(x)$, $0 \le x \le 1$, $f(0) = 0$. For a point P on C_1, let the lines through P, parallel to the axes, meet C_2 & C_3 at Q & R respectively (see figure). If for every position of P (on C_1), the areas of the shaded regions OPQ & ORP are equal, determine the function f(x). [JEE '98, 8]



Q.4 Indicate the region bounded by the curves $y = x\,ln\,x$ & $y = 2x - 2x^2$ and obtain the area enclosed by them. [REE '98, 6]

Q.5

(a) For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals 9/2 ?

(A) – 4 (B) – 2 (C) 2 (D) 4

(b) Let f(x) be a continuous function given by $f(x) = \begin{cases} 2x & \text{for } |x| \le 1 \\ x^2 + ax + b & \text{for } |x| > 1 \end{cases}$

Find the area of the region in the third quadrant bounded by the curves, $x = -2y^2$ and

$y = f(x)$ lying on the left of the line $8x + 1 = 0$. [JEE '99, 3 + 10 (out of 200)]

Q.6 Find the area of the region lying inside $x^2 + (y-1)^2 = 1$ and outside $c^2x^2 + y^2 = c^2$ where $c = \sqrt{2} - 1$. [REE '99, 6]

Q.7 Find the area enclosed by the parabola $(y-2)^2 = x - 1$, the tangent to the parabola at (2, 3) and the x-axis. [REE 2000,3]

Q.8 Let $b \neq 0$ and for $j = 0, 1, 2, \ldots\ldots\ldots n$, let S_j be the area of the region bounded by the y axis and the curve $xe^{ay} = \sin by$, $\frac{j\pi}{b} \leq y \leq \frac{(j+1)\pi}{b}$. Show that $S_0, S_1, S_2, \ldots\ldots\ldots\ldots S_n$ are in geometric progression. Also, find their sum for $a = -1$ and $b = \pi$. [JEE'2001, 5]

Q.9 The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

(A) 1 (B) 2 (C) $2\sqrt{2}$ (D) 4

[JEE'2002, (Scr)]

Q.10 Find the area of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and $y = 2$, which lies to the right of the line $x = 1$. [JEE '2002, (Mains)]

Q.11 If the area bounded by $y = ax^2$ and $x = ay^2$, $a > 0$, is 1, then a =

(A) 1 (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{3}$ (D) $-\frac{1}{\sqrt{3}}$

[JEE '2004, (Scr)]

Q.12(a) The area bounded by the parabolas $y = (x + 1)^2$ and $y = (x - 1)^2$ and the line $y = 1/4$ is

(A) 4 sq. units (B) 1/6 sq. units (C) 4/3 sq. units (D) 1/3 sq. units

[JEE '2005 (Screening)]

(b) Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$.

(c) If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$, $f(x)$ is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of $y = f(x)$ with x-axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by $f(x)$ and chord AB. [JEE '2005 (Mains), 4 + 6]

Q.13 Match the following

(i) $\int_0^{\pi/2} (\sin x)^{\cos x} (\cos x \cot x - \log(\sin x)^{\sin x})\, dx$ (A) 1

(ii) Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$ (B) 0

(iii) Cosine of the angle of intersection of curves $y = 3^{x-1} \log x$ and $y = x^x - 1$ is (C) $6 \ln 2$

(D) 4/3 [JEE 2006, 6]

ANSWER
EXERCISE–I

Q.1 5/6 sq. units **Q 2.** 21/2 sq. units **Q 3.** 23/6 sq. units

Q 4. $c = -\frac{\pi}{6}$ or $\frac{\pi}{3}$ **Q 5.** $x_0 = 2$, $A(x_0) = 8$ **Q 6.** $(e^2 - 5)/4e$ sq. units

Q 7. $\pi - \tan^{-1}\frac{2\sqrt{2}}{3\pi}$; $\pi - \tan^{-1}\frac{4\sqrt{2}}{3\pi}$ **Q 8.** $\frac{11}{8}$ sq. units

Q 9. $\frac{\pi}{2}$; $\frac{\pi - 1}{\pi + 1}$ **Q 10.** a = 9 **Q 11.** $\frac{3\pi + 2}{\pi - 2}$

Q 12. $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ sq. units **Q 13.** **(i)** m = 1, **(ii)** m = ∞ ; $A_{min} = 4/3$

Q 14. $\sqrt{2}$ **Q 15.** 2 sq. units **Q 16.** $\pi/4$

Q 17. a=– 3/4 **Q 18.** $\sqrt{3}$ **Q 19.** $\frac{1}{3} + \ell n\left(\frac{\sqrt{3}}{2}\right)$ sq. units

Q 21. $1 - 3e^{-2}$ **Q 22.** C = – 1 or $\left(8 - \sqrt{17}\right)^{1/3}$

Q 23. 1/3 **Q 24.** $\frac{1}{2}(1 - e^{-1/2})$ **Q 25.** $ln\,2$

EXERCISE–II

Q 1. 4 : 121 **Q 2.** 128/15 sq. units **Q 3.** $(5\pi - 2)/4$ sq. units

Q 4. $y = 2x/3$ **Q 5.** $\sqrt{2}+1$ **Q 6.** b = 1/8, $A_{minimum} = 4\sqrt{3}$ sq. units

Q 7. f(x) = x sinx, a = 1; $A_1 = 1 - \sin 1$; $A_2 = \pi - 1 - \sin 1$; $A_3 = (3\pi - 2)$ sq. units

Q 8. $a = 1 + e^2$, $b = 1 + e^{-2}$ **Q.9** 1/2 **Q.10** a = 8 or $\frac{2}{5}\left(6 - \sqrt{21}\right)$

Q 11. $2 - (\pi/2)$ sq. units **Q 12.** K = 2 , A = 32/3 **Q.14** $a = \frac{2}{3}$

Q 15. $\alpha = \pi/3$, ratio = $2 : \sqrt{3}$ **Q 16.** $\frac{8}{3} - \frac{8}{3}\left(3-2\sqrt{2}\right)^{3/2} - \left(2\sqrt{2} - 2\right)^2$

Q 17. $\left[\frac{\sqrt{2}}{3} + \frac{3}{2}.\,\text{arc sin}\,\frac{1}{3}\right]a^2$ sq. units **Q 18.** $\frac{1}{3}\left(16\sqrt{2} - 20\right)$ **Q 19.** $4a^2$

Q 20. a = 1/2 gives minima, $A\left(\frac{1}{2}\right) = \frac{3\sqrt{3} - \pi}{12}$; a = 0 gives local maxima $A(0) = 1 - \frac{\pi}{4}$;
a = 1 gives maximum value, $A(1) = \pi/4$

Q 21. $\frac{32}{3} - 4\sqrt{3} + \frac{8\pi}{3}$ **Q 22.** $3\left(\sqrt{3} - 1\right)$ sq. units **Q 23.** $(16/9)\,x^2$

Q 24. for a = 1, area is greatest, for a = 1/2, area is least **Q25.** $e^{\pi/3}\log 2$ sq. units

EXERCISE–III

Q.1 17/27 **Q.2** 5/6 sq. units **Q.3** $f(x) = x^3 - x^2$ **Q.4** 7/12

Q.5 **(a)** B, D **(b)** 257/192 ; a = 2 ; b = – 1 **Q.6** $\left(\pi - \frac{\pi - 2}{2\sqrt{2}}\right)$ sq. units

Q.7 9 sq. units **Q.8** $\frac{S_j}{S_{j+1}} = e^{\frac{\pi a}{b}}$; $S_0 = \frac{b\left(e^{-\frac{a\pi}{b}} + 1\right)}{a^2 + b^2}$ for a = –1, b = π, $S_0 = \frac{\pi(e+1)}{\pi^2 + 1}$ and r = π

Q.9 B **Q.10** $\left(\frac{20}{3} - 4\sqrt{2}\right)$ sq. units **Q.11** B

Q.12 (a) D ; (b) $\frac{1}{3}$ sq. units ; (c) $\frac{125}{3}$ sq. units **Q.13** **(i)** A, (ii) D, (iii) A

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EXERCISE–IV

1. The area bounded by the curve $x^2 = 4y$, x-axis and the line x = 2 is

(A) 1 (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) 2

2. The area bounded by the x-axis and the curve $y = 4x - x^2 - 3$ is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{8}{3}$

3. The area bounded by the curve y = sin ax with x-axis in one arc of the curve is

(A) $\frac{4}{a}$ (B) $\frac{2}{a}$ (C) $\frac{1}{a}$ (D) 2a

4. The area contained between the curve $xy = a^2$, the vertical line x = a, x = 4a (a > 0) and x-axis is

(A) $a^2 \log 2$ (B) $2a^2 \log 2$ (C) a log 2 (D) 2a log 2

5. The area of the closed figure bounded by the curves $y = \sqrt{x}$, $y = \sqrt{4-3x}$ & y = 0 is:

(A) $\frac{4}{9}$ (B) $\frac{8}{9}$ (C) $\frac{16}{9}$ (D) none

6. The area of the closed figure bounded by the curves y = cos x; $y = 1+\frac{2}{\pi}x$ & $x = \frac{\pi}{2}$ is

(A) $\frac{\pi+4}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{3\pi+4}{4}$ (D) $\frac{3\pi-4}{4}$

7. The area included between the curve $xy^2 = a^2(a-x)$ & its asymptote is:

(A) $\frac{\pi a^2}{2}$ (B) $2\pi a^2$ (C) πa^2 (D) none

8. The area bounded by $x^2 + y^2 - 2x = 0$ & $y = \sin\frac{\pi x}{2}$ in the upper half of the circle is:

(A) $\frac{\pi}{2} - \frac{4}{\pi}$ (B) $\frac{\pi}{4} - \frac{2}{\pi}$ (C) $\pi - \frac{8}{\pi}$ (D) none

9. The area of the region enclosed between the curves $7x^2 + 9y + 9 = 0$ and $5x^2 + 9y + 27 = 0$ is:

(A) 2 (B) 4 (C) 8 (D) 16

10. The area bounded by the curves y = x (1 – ln x); $x = e^{-1}$ and a positive X–axis between $x = e^{-1}$ and x = e is :

(A) $\left(\frac{e^2 - 4e^{-2}}{5}\right)$ (B) $\left(\frac{e^2 - 5e^{-2}}{4}\right)$ (C) $\left(\frac{4e^2 - e^{-2}}{5}\right)$ (D) $\left(\frac{5e^2 - e^{-2}}{4}\right)$

11. The area enclosed between the curves $y = \log_e(x + e)$, $x = \log_e\left(\frac{1}{y}\right)$ and the x-axis is

(A) 2 (B) 1 (C) 4 (D) none of these

12. The area bounded by the curves $\sqrt{x} + \sqrt{y} = 1$ and x + y = 1 is

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) none of these

13. The area bounded by x-axis, curve y = f(x), and lines x = 1, x = b is equal to $\sqrt{(b^2+1)} - \sqrt{2}$ for all b > 1, then f(x) is

(A) $\sqrt{(x-1)}$ (B) $\sqrt{(x+1)}$ (C) $\sqrt{(x^2+1)}$ (D) $x/\sqrt{(1+x^2)}$

14. The area of the region for which $0 < y < 3 - 2x - x^2$ and x > 0 is

(A) $\int_1^3 (3-2x-x^2)\,dx$ (B) $\int_0^3 (3-2x-x^2)\,dx$ (C) $\int_0^1 (3-2x-x^2)\,dx$ (D) $\int_{-1}^3 (3-2x-x^2)\,dx$

15. The area bounded by $y = x^2$, y = [x + 1], $x \le 1$ and the y-axis is
(A) 1/3 (B) 2/3 (C) 1 (D) 7/3

16. The area bounded by the curve $x = a\cos^3 t$, $y = a\sin^3 t$ is

(A) $\frac{3\pi a^2}{8}$ (B) $\frac{3\pi a^2}{16}$ (C) $\frac{3\pi a^2}{32}$ (D) $3\pi a^2$

17. If A_1 is the area enclosed by the curve xy = 1, x -axis and the ordinates x = 1, x = 2; and A_2 is the area enclosed by the curve xy = 1, x -axis and the ordinates x = 2, x = 4, then
(A) $A_1 = 2A_2$ (B) $A_2 = 2A_1$ (C) $A_2 = 2A_1$ (D) $A_1 = A_2$

18. The area bounded by the curve y = f(x), x-axis and the ordinates x = 1 and x = b is (b – 1) sin (3b + 4), $\forall$ b $\in$ R, then f(x) =
(A) (x – 1) cos (3x + 4) (B) sin (3x + 4)
(C) sin (3x + 4) + 3(x – 1) cos (3x + 4) (D) none of these

19. Find the area of the region bounded by the curves $y = x^2 + 2$, y = x, x = 0 and x = 3.

(A) $\frac{21}{2}$ sq. unit (B) 22 sq. unit (C) 21 sq. unit (D) none of these

20. The areas of the figure into which curve $y^2 = 6x$ divides the circle $x^2 + y^2 = 16$ are in the ratio

(A) $\frac{2}{3}$ (B) $\frac{4\pi-\sqrt{3}}{8\pi+\sqrt{3}}$ (C) $\frac{4\pi+\sqrt{3}}{8\pi-\sqrt{3}}$ (D) none of these

21. The triangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point (1, 1) and the coordinate axes, lies in the first quadrant . If its area is 2, then the value of b is **[IIT - 2001]**
(A) – 1 (B) 3 (C) – 3 (D) 1

EXERCISE–V

1. Find the area of the region bounded by the curve $y^2 = 2y - x$ and the y-axis.

2. Find the value of c for which the area of the figure bounded by the curves y = sin 2x, the straight lines x = π/6, x = c & the abscissa axis is equal to 1/2.

3. For what value of 'a' is the area bounded by the curve $y = a^2x^2 + ax + 1$ and the straight line y = 0, x = 0 & x = 1 the least?

4. Find the area of the region bounded in the first quadrant by the curve C: y = tan x, tangent drawn to

C at $x = \frac{\pi}{4}$ and the x – axis.

5. Find the values of m (m > 0) for which the area bounded by the line y = mx + 2 and $x = 2y - y^2$ is, (i) 9/2 square units & (ii) minimum. Also find the minimum area.

6. Consider the two curves $y = 1/x^2$ & $y = 1/[4(x - 1)]$.

(i) At what value of 'a' (a > 2) is the reciprocal of the area of the figure bounded by the curves, the lines x = 2 & x = a equal to 'a' itself?

(ii) At what value of 'b' (1 < b < 2) the area of the figure bounded by these curves, the lines x = b & x = 2 equal to 1 – 1/b.

7. A normal to the curve, $x^2 + \alpha x - y + 2 = 0$ at the point whose abscissa is 1, is parallel to the line y = x. Find the area in the first quadrant bounded by the curve, this normal and the axis of ' x '.

8. Find the area between the curve $y^2(2a - x) = x^3$ & its asymptotes.

9. Draw a neat & clean graph of the function $f(x) = \cos^{-1}(4x^3 - 3x)$, $x \in [-1, 1]$ & find the area enclosed between the graph of the function & the x–axis as x varies from 0 to 1.

10. Find the area of the loop of the curve, $ay^2 = x^2(a - x)$.

11. Let $b \neq 0$ and for j = 0, 1, 2,......, n, let S_j be the area of the region bounded by the y–axis and the curve $xe^{ay} = \sin by$, $\frac{j\pi}{b} \le y \le \frac{(j+1)\pi}{b}$. Show that $S_0, S_1, S_2, \ldots, S_n$ are in geometric progression. Also, find their sum for a = – 1 and b = π. **[IIT - 2001, 5]**

12. Find the area of the region bounded by the curves, $y = x^2$, $y = |2 - x^2|$ & y = 2 which lies to the right of the line x = 1. **[IIT - 2002, 5]**

13. If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$, f(x) is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of y = f(x) with x-axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by f(x) and cheord AB. **[IIT - 2005, 6]**

ANSWER

EXERCISE–IV

1. B **2.** C **3.** B **4.** B **5.** B **6.** D **7.** C

8. A **9.** C **10.** B **11.** A **12.** A **13.** D **14.** C

15. B **16.** A **17.** D **18.** C **19.** A **20.** C **21.** C

EXERCISE–V

1. 4/3 sq. units **2.** $c = -\frac{\pi}{6}$ or $\frac{\pi}{3}$ **3.** $a = -\frac{3}{4}$

4. $\frac{1}{2}\ln 2 - \frac{1}{4}$ **5.** (i) m = 1, (ii) m = ∞; $A_{min} = 4/3$

6. $a = 1 + e^2$, $b = 1 + e^{-2}$ **7.** $\frac{7}{6}$ **8.** $3\pi a^2$

9. $3\left(\sqrt{3} - 1\right)$ sq. units **10.** $\frac{8a^2}{15}$

11. $\frac{20}{3} - 4\sqrt{2}$ sq. units **13.** $\frac{125}{3}$ square units.